ANALIZA MATEMATYCZNA

LISTA ZADAŃ 12

6.01.14

(1) Podaj wzór na $C_n = \sum_{i=1}^{n} \frac{b-a}{n} f\big(a + i\,\frac{b-a}{n}\big)$, a następnie oblicz $\lim_{n\to\infty} C_n$

(a) $f(x) = 1,\ a = 5,\ b = 8,$ (b) $f(x) = x,\ a = 0,\ b = 1,$
(c) $f(x) = x,\ a = 1,\ b = 5,$ (d) $f(x) = x^2,\ a = 0,\ b = 5,$
(e) $f(x) = x^3,\ a = 0,\ b = 1,$ (f) $f(x) = 2x + 5,\ a = -3,\ b = 4,$
(g) $f(x) = x^2 + 1,\ a = -1,\ b = 2,$ (h) $f(x) = x^3 + x,\ a = 0,\ b = 4,$
(i) $f(x) = e^x,\ a = 0,\ b = 1.$

(2) Oblicz następujące całki oznaczone poprzez konstrukcję ciągu podziałów przedziału, odpowiadającego mu ciągu sum Riemanna, oraz jego granicy

(a) $\int_2^4 x^{10}\,dx,\ (x_i = 2\cdot 2^{i/n}),$ (b) $\int_1^e \frac{\log(x)}{x}\,dx,\ (x_i = e^{i/n}),$
(c) $\int_0^{20} x\,dx,$ (d) $\int_1^{10} e^{2x}\,dx,$
(e) $\int_0^1 \sqrt[3]{x}\,dx,\ (x_i = \frac{i^3}{n^3}),$ (f) $\int_{-1}^1 |x|\,dx,$
(g) $\int_1^2 \frac{dx}{x}\,dx,\ (x_i = 2^{i/n}),$ (h) $\int_0^4 \sqrt{x}\,dx,\ (x_i = \frac{4i^2}{n^2}).$

(3) Oblicz całki oznaczone

(a) $\int_{-\pi}^{\pi} \sin(x^{2007})\,dx,$ (b) $\int_0^2 \arctan([x])\,dx,$
(c) $\int_0^2 [\cos(x^2)]\,dx,$ (d) $\int_0^1 \sqrt{1+x}\,dx,$
(e) $\int_{-2}^{-1} \frac{1}{(11+5x)^3}\,dx,$ (f) $\int_{-13}^{2} \frac{1}{\sqrt[5]{(3-x)^4}}\,dx,$
(g) $\int_0^1 \frac{x}{(x^2+1)^2}\,dx,$ (h) $\int_0^3 \operatorname{sgn}(x^3 - x)\,dx,$
(i) $\int_0^1 x\,e^{-x}\,dx,$ (j) $\int_0^{\pi/2} x\cos(x)\,dx,$
(k) $\int_0^{e-1} \log(x+1)\,dx,$ (l) $\int_0^{\pi} x^3\,\sin(x)\,dx,$
(m) $\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1}\,dx,$ (n) $\int_1^{e^3} \frac{1}{x\sqrt{1+\log(x)}}\,dx,$
(o) $\int_1^2 \frac{1}{x+x^3}\,dx,$ (p) $\int_0^2 \frac{1}{\sqrt{x+1}+\sqrt{(x+1)^3}}\,dx,$
(q) $\int_0^5 |x^2 - 5x + 6|\,dx,$ (r) $\int_0^1 \frac{e^x}{e^x - e^{-x}}\,dx,$

(s) $\displaystyle\int_1^2 x\,\log_2(x)\,dx,$ (t) $\displaystyle\int_0^{\sqrt{7}} \frac{x^3}{\sqrt[3]{1+x^2}}\,dx,$

(u) $\displaystyle\int_0^{6\pi} |\sin(x)|\,dx,$ (w) $\displaystyle\int_0^{\pi/2} \cos(x)\sin^{11}(x)\,dx,$

(x) $\displaystyle\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+5}\,dx,$ (y) $\displaystyle\int_{-\pi}^{\pi} x^{2007}\cos(x)\,dx,$

(z) $\displaystyle\int_0^{2\pi} (x-\pi)^{2007}\cos(x)\,dx.$

(4) Udowodnij następujące oszacowania

(a) $\displaystyle\int_0^{\pi/2} \frac{\sin(x)}{x}\,dx < 2,$ (b) $\displaystyle\frac{1}{5} < \int_1^2 \frac{1}{x^2+1}\,dx < \frac{1}{2},$

(c) $\displaystyle\frac{1}{11} < \int_9^{10} \frac{1}{x+\sin(x)}\,dx < \frac{1}{8},$ (d) $\displaystyle\int_{-1}^{2} \frac{|x|}{x^2+1}\,dx < \frac{3}{2},$

(e) $\displaystyle\int_0^1 x(1-x^{99+x})\,dx < \frac{1}{2},$ (f) $\displaystyle 2\sqrt{2} < \int_2^4 x^{1/x}\,dx,$

(g) $\displaystyle 5 < \int_1^3 x^x\,dx < 31,$ (h) $\displaystyle\int_1^2 \frac{1}{x}\,dx < \frac{3}{4}.$

(5) Oblicz następujące granice

(a) $\lim\limits_{n\to\infty} \left(\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}\right),$

(b) $\lim\limits_{n\to\infty} \left(\frac{1^{20}+2^{20}+3^{20}+\cdots+n^{20}}{n^{21}}\right),$

(c) $\lim\limits_{n\to\infty} \left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \cdots + \frac{1}{(2n)^2}\right)\cdot n,$

(d) $\lim\limits_{n\to\infty} \left(\frac{1}{\sqrt{n}\sqrt{2n}} + \frac{1}{\sqrt{n}\sqrt{2n+1}} + \frac{1}{\sqrt{n}\sqrt{2n+2}} + \frac{1}{\sqrt{n}\sqrt{2n+3}} + \cdots + \frac{1}{\sqrt{n}\sqrt{3n}}\right),$

(e) $\lim\limits_{n\to\infty} \left(\sin(\frac{1}{n}) + \sin(\frac{2}{n}) + \sin(\frac{3}{n}) + \cdots + \sin(\frac{n}{n})\right)\cdot\frac{1}{n},$

(f) $\lim\limits_{n\to\infty} \left(\sqrt{4n} + \sqrt{4n+1} + \sqrt{4n+2} + \cdots + \sqrt{5n}\right)\cdot\frac{1}{n\sqrt{n}},$

(g) $\lim\limits_{n\to\infty} \left(\frac{1}{\sqrt[3]{n}} + \frac{1}{\sqrt[3]{n+1}} + \frac{1}{\sqrt[3]{n+2}} + \cdots + \frac{1}{\sqrt[3]{8n}}\right)\cdot\frac{1}{\sqrt[3]{n^2}},$

(h) $\lim\limits_{n\to\infty} \left(\frac{\sqrt[6]{n}\cdot(\sqrt[3]{n}+\sqrt[3]{n+1}+\sqrt[3]{n+2}+\cdots+\sqrt[3]{2n})}{\sqrt{n}+\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{2n}}\right),$

(i) $\lim\limits_{n\to\infty} \left(\frac{n}{n^2} + \frac{n}{n^2+1} + \frac{n}{n^2+4} + \frac{n}{n^2+9} + \frac{n}{n^2+16} + \cdots + \frac{n}{n^2+n^2}\right),$

(j) $\lim\limits_{n\to\infty} \left(\frac{4}{5n} + \frac{4}{5n+3} + \frac{4}{5n+6} + \frac{4}{5n+9} + \cdots + \frac{4}{26n}\right),$

(k) $\lim\limits_{n\to\infty} \left(\frac{1}{7n} + \frac{1}{7n+2} + \frac{1}{7n+4} + \frac{1}{7n+6} + \cdots + \frac{1}{9n}\right),$

(l) $\lim\limits_{n\to\infty} \left(\frac{1}{7n^2} + \frac{1}{7n^2+1} + \frac{1}{7n^2+2} + \frac{1}{7n^2+3} + \cdots + \frac{1}{8n^2}\right),$

(m) $\lim\limits_{n\to\infty} \frac{1}{n}\left(e^{\sqrt{\frac{1}{n}}} + e^{\sqrt{\frac{2}{n}}} + e^{\sqrt{\frac{3}{n}}} + \cdots + e^{\sqrt{\frac{n}{n}}}\right),$

(n) $\lim\limits_{n\to\infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+3}} + \frac{1}{\sqrt{n+6}} + \frac{1}{\sqrt{n+9}} + \cdots + \frac{1}{\sqrt{7n}}\right)\frac{1}{\sqrt{n}},$

(o) $\lim\limits_{n\to\infty} \left(\frac{n^2+0}{(3n)^3} + \frac{n^2+1}{(3n+1)^3} + \frac{n^2+2}{(3n+2)^3} + \frac{n^2+3}{(3n+3)^3} + \cdots + \frac{n^2+n}{(4n)^3}\right),$

(p) $\lim\limits_{n\to\infty} \left(\frac{n}{2n^2} + \frac{n}{2(n+1)^2} + \frac{n}{2(n+2)^2} + \frac{n}{2(n+3)^2} + \cdots + \frac{n}{50n^2}\right),$

(r) $\lim\limits_{n\to\infty} \left(\frac{n}{2n^2} + \frac{n}{n^2+(n+1)^2} + \frac{n}{n^2+(n+2)^2} + \frac{n}{n^2+(n+3)^2} + \cdots + \frac{n}{50n^2}\right).$